Equivariant Deep Learning of Mixed-Integer Optimal Control Solutions for Vehicle Decision Making and Motion Planning

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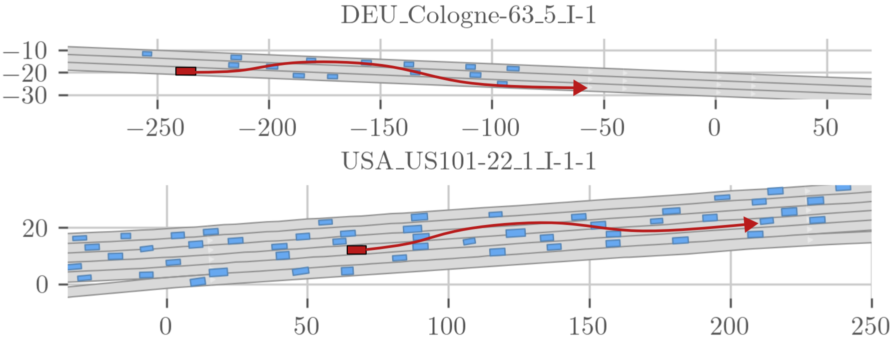
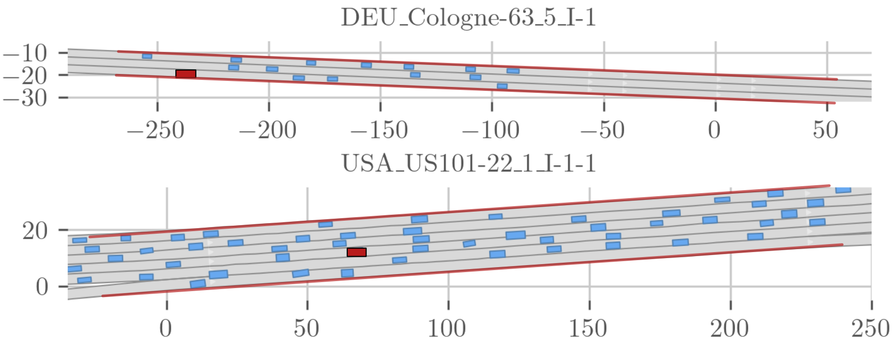
Group Reatreat
SYSCOP
June 10, 2024

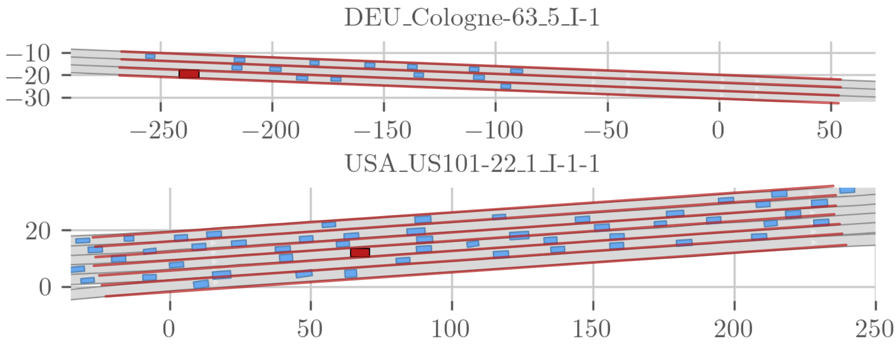
Task: Motion Planning on Highways



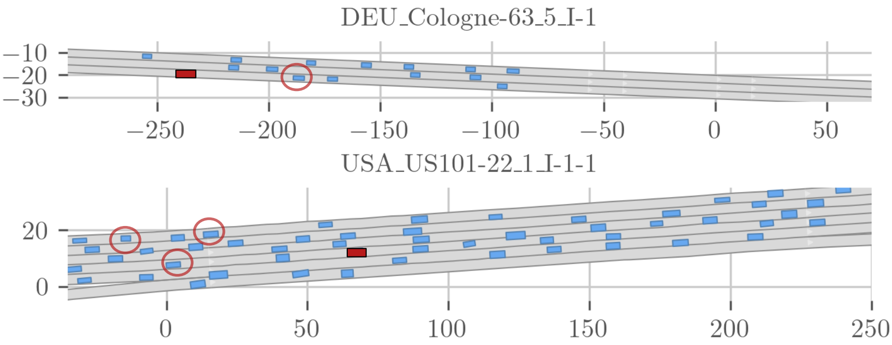
mostly straight



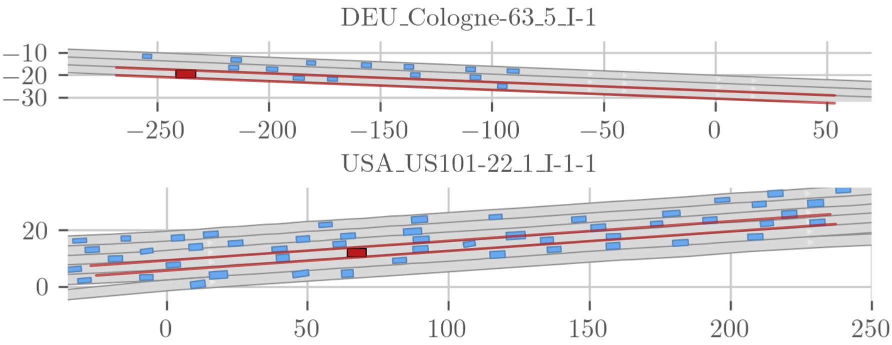
parallel lanes



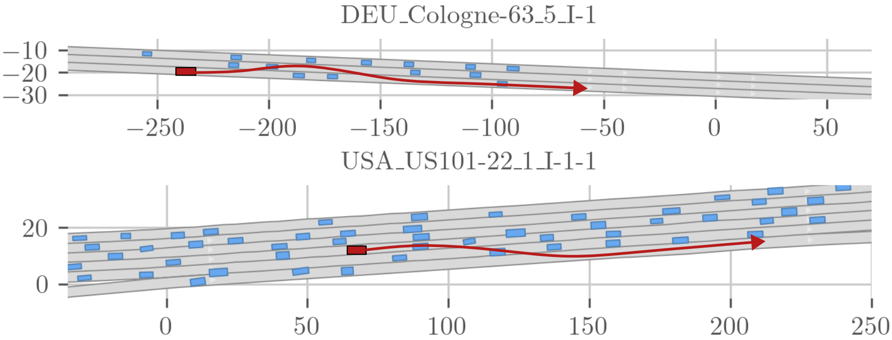
multiple similar obstacles



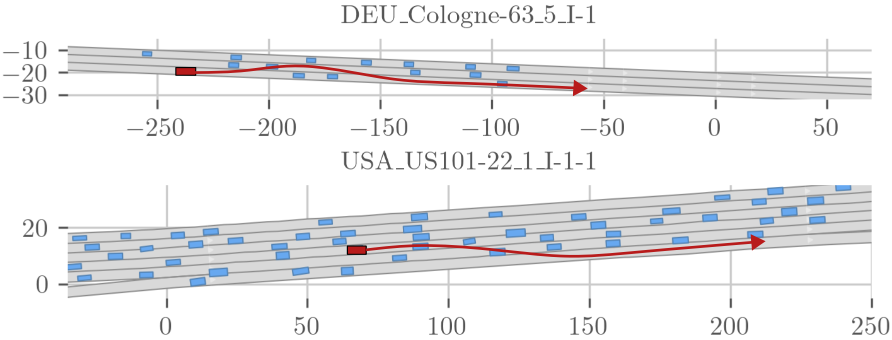
rules: lane keeping



rules: keep right, speed limit



objective: set speed, set lane



$$
\begin{aligned}
\min_{\substack{X\in\mathbb{R}^{n_x\times N},\\ U\in\mathbb{R}^{n_u\times N-1},\\ \beta\in\{0,1\}^{N_\mathrm{b}}}} \quad & J(X,U,\beta) \\
\text{s.t.} \quad & \\
& x_0 = \hat{x} \\
& x_{i+1} = Ax_i + Bu_i, \quad i = 1,\ldots,N-1, \\
& H(X,U) \geq 0, \\
& H_{\mathrm{bin}}(X,\beta) \geq 0
\end{aligned}
$$

[1] Rien Quirynen, Sleiman Safaoui, and Stefano Di Cairano. "Real-time Mixed-Integer Quadratic Programming for Vehicle Decision Making and Motion Planning". In: *ArXiv* (2023). arXiv: 2308.10069 [math.OC].

- Slow online computation time
- MIQP solvers are not usual for embedded hardware
- MIQP solvers are expensive

$$
\begin{aligned}
\min_{\substack{X \in \mathbb{R}^{n_x \times N},\\ U \in \mathbb{R}^{n_u \times N-1},\\ \beta \in \{0,1\}^{N_\mathrm{b}}}} \quad & J(X, U, \beta) \\
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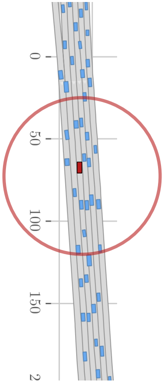
\end{aligned}
$$

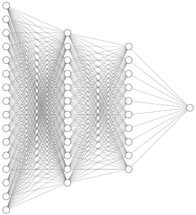
Introduction

Speed-up computation time

learn binary assignments through simulation by supervised learning

$\rightarrow$ only solve QP $\rightarrow$ much faster than solving MIQP





$$
\begin{aligned}
\min_{\substack{X \in \mathbb{R}^{n_x \times N},\\ U \in \mathbb{R}^{n_u \times N-1}}} \quad & J(X, U; \beta) \\
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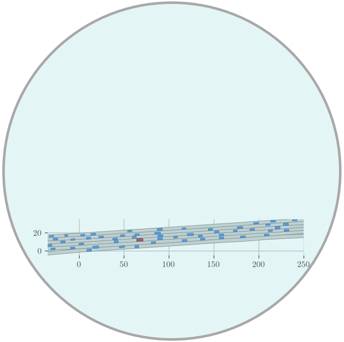
$$

motion planning on highways in autonomous driving stack



AD
Motion
Plannning

mixed integer optimization problem formulation

Mixed-Integer Optimization

$$
\begin{aligned}
\min_{\substack{X\in\mathbb{R}^{n_x\times N},\\ U\in\mathbb{R}^{n_u\times N-1},\\ \beta\in\{0,1\}^{N_\mathrm{b}}}} \quad & J(X,U,\beta) \\
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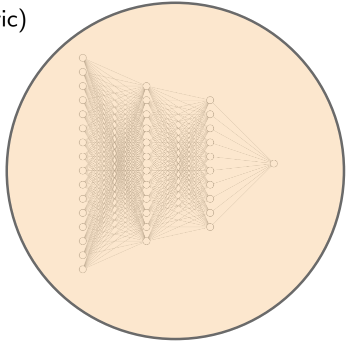
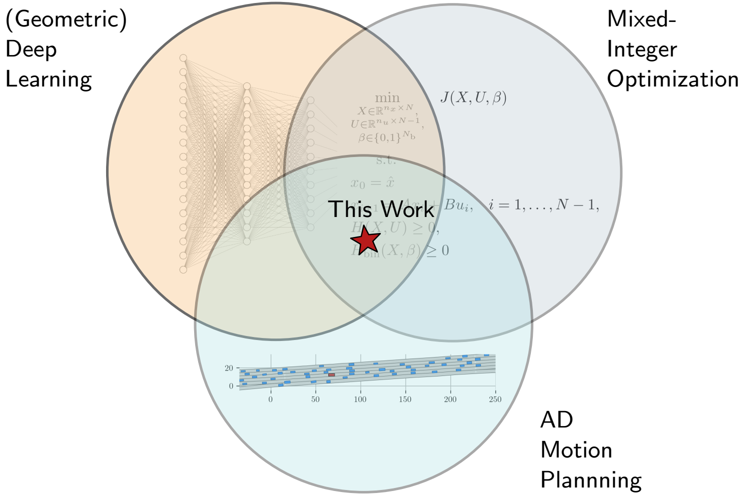
\end{aligned}
$$

structure-exploiting neural network architecture

(Geometric)
Deep
Learning





(Geometric) Deep Learning
Mixed-Integer Optimization
AD Motion Plannning
This Work
$\min_{X \in \mathbb{R}^{n_x \times N}, U \in \mathbb{R}^{n_u \times N-1}, \beta \in \{0,1\}^{N_b}} J(X,U,\beta)$
s.t.
$x_0 = \hat{x}$
$Bu_i, \quad i = 1, \ldots, N-1,$
$H(X,U) \geq 0,$
≥ 0
20
0
0
50
100
150
200
250

Outline

1. Mixed-Integer Problem Formulation
2. Learning Binary Variables
3. Geometric Deep Learning
4. Additional Concepts
5. Simulation Results

$$
\begin{aligned}
\min_{\substack{X\in\mathbb{R}^{n_x\times N},\\ U\in\mathbb{R}^{n_u\times N-1},\\ \beta\in\{0,1\}^{N_\mathrm{b}}}} \quad & J(X,U,\beta)\\
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$$

Mixed-Integer Optimization

1. Mixed-Integer Problem Formulation

Two categories of binary variables:

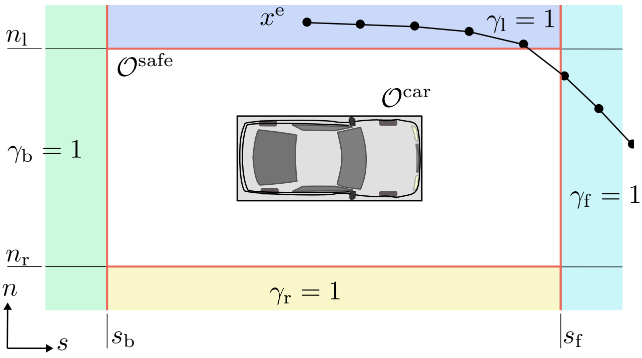
- Expression of nonconvex configuration space as a disjunction of convex sets
- Choice of lane

1. Mixed-Integer Problem Formulation

Two categories of binary variables:

- Expression of nonconvex configuration space as a disjunction of convex sets (**Why?**)
- Choice of lane

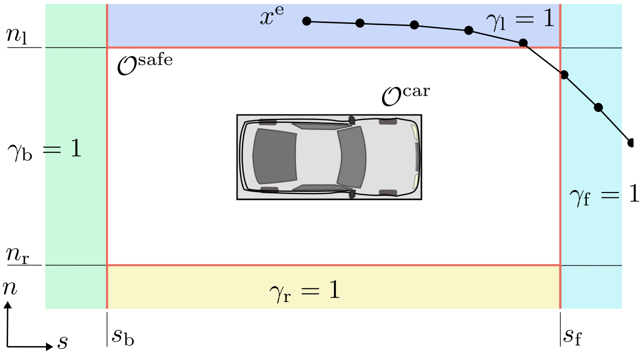
- Over-approximating obstacle $\mathcal{O}^{\mathrm{car}}$ by $\mathcal{O}^{\mathrm{safe}}$



1. Mixed-Integer Problem Formulation

Nonconvex free configuration space

- Over-approximating obstacle $\mathcal{O}^{\mathrm{car}}$ by $\mathcal{O}^{\mathrm{safe}}$
- Split configuration space $\mathcal{F} = \mathbb{R}^2 \backslash \mathcal{O}^{\mathrm{safe}}$ into 4 convex sets (left, right, front, back)



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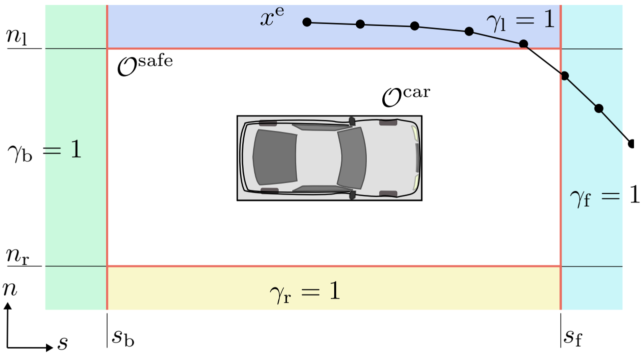
- Split configuration space $\mathcal{F} = \mathbb{R}^2 \backslash \mathcal{O}^{\mathrm{safe}}$ into 4 convex sets (left, right, front, back)
- Assign binary indicator variables $\gamma_{\mathrm{l}}, \gamma_{\mathrm{r}}, \gamma_{\mathrm{f}}, \gamma_{\mathrm{b}} \in \{0, 1\}$



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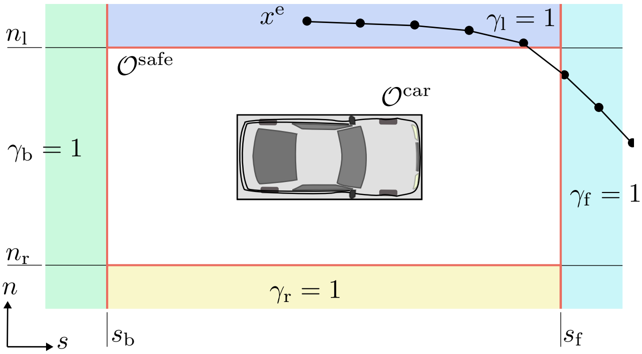
- Assign binary indicator variables $\gamma_{\mathrm{l}}, \gamma_{\mathrm{r}}, \gamma_{\mathrm{f}}, \gamma_{\mathrm{b}} \in \{0, 1\}$
- 4 binary variables per obstacle per time step: $4N_{\mathrm{obs}}N$ binary variables



- Adding reference state as decision variable $\tilde{n}$, with $\tilde{X}_{\mathrm{n}} = [\tilde{n}_0, \dots, \tilde{n}_N]^\top$
- Adding binary lane change control variables $\lambda^{\mathrm{up}}, \lambda^{\mathrm{down}}$
- Reference dynamics:

$$\tilde{n}_{i+1} = \tilde{n}_i + d_{\mathrm{lane}} \lambda_i^{\mathrm{up}} - d_{\mathrm{lane}} \lambda_i^{\mathrm{down}}$$

- Tracking cost for lateral state n:

$$\sum_{i=0}^{N} w_n (\tilde{n}_i - n_i)^2$$

- adding $2N$ lane change binary variables to a total

$$N_{\mathrm{bin}} = 2N + 4N N_{\mathrm{obs}}$$

1. Mixed-Integer Problem Formulation

$$\min_{\substack{X\in\mathbb{R}^{n_x\times N}, U\in\mathbb{R}^{n_u\times N-1},\\ \tilde{X}_n\in\mathbb{R}^{N},\\ \Gamma\in\{0,1\}^{4NN_{\text{obs}}},\\ \Lambda\in\{0,1\}^{2N}}} \quad J(X,U,\tilde{X}_n)$$

$$\text{s.t.}$$

$$\begin{aligned}
& x_0 = \hat{x}, \quad \tilde{n}_0 = \hat{n}_0, && \\
& x_{i+1} = Ax_i + Bu_i, && i = 0,\dots,N-1, \\
& \tilde{n}_{i+1} = \tilde{n}_i + d_{\text{lane}}\lambda_i^{\text{up}} - d_{\text{lane}}\lambda_i^{\text{down}}, && i = 0,\dots,N-1, \\
& H(X,U) \geq 0, && \\
& H_{\text{obs}}(x_i, (\gamma_{\text{d}})_{i,j}) \geq 0 && i = 0,\dots,N-1, \\
& && j = 1,\dots,N_{\text{obs}}
\end{aligned}$$

2. Learning Binary Variables

Generate training data and class labels

- Randomize problem features p_i
  - ego state
  - obstacle states
  - vehicle dimensions
  - road geometry
- Solve MIQPs to obtain binary assignments $\beta_i^\star = (\Gamma_i^\star, \Lambda_i^\star)$
- If $\beta_i^\star$ not in data set $\mathcal{D}$: generate class label l_i and add $(p_i, l_i, \beta_i^\star)$ to $\mathcal{D}$
- If $\beta_i^\star$ in data set $\mathcal{D}$: use already existing label l_j, with $\beta_i^\star = \beta_j^\star$ and add $(p_i, l_j, \beta_j^\star)$ to $\mathcal{D}$

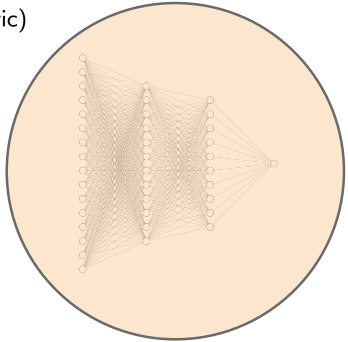
Use data set $\mathcal{D}$ to train a classifier that predicts $\beta^\star$

- Labels learned by classification as opposed to regression
- Number of assignments in theory $2^{4NN_{\mathrm{obs}}+2N}$
- If assignment was never seen in data, no label exists
- Shown to perform better than regression[2]

[2] Dimitris Bertsimas and Bartolomeo Stellato. "The voice of optimization". en. In: *Machine Learning* 110.2 (Feb. 2021), pp. 249–277. ISSN: 1573-0565. DOI: 10.1007/s10994-020-05893-5.

3. Geometric Deep Learning

(Geometric)
Deep
Learning



3. Geometric Deep Learning

Geometric Deep Learning
Grids, Groups, Graphs,
Geodesics, and Gauges

Michael M. Bronstein[1], Joan Bruna[2], Taco Cohen[3], Petar Veličković[4]

May 4, 2021

3. Geometric Deep Learning

Fundamentally, geometric deep learning involves encoding a geometric understanding of data as an inductive bias in deep learning models to give them a helping hand.

Definition

Let $f(x) : \mathbb{X}^M \to \mathbb{Y}$ be a function on a set of variables $x = \{x_1, \dots, x_M\} \in \mathbb{X}^M$ and let $\mathcal{G}$ be the permutation group on $\{1, \dots, M\}$. The function f is **permutation invariant**, if $f(g \cdot x) = f(x)$ for all $g \in \mathcal{G}, x \in \mathbb{X}^M$.

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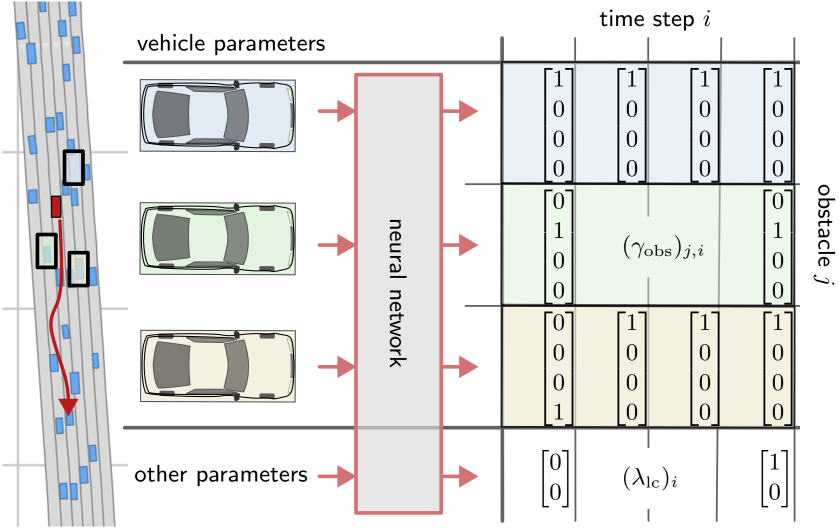
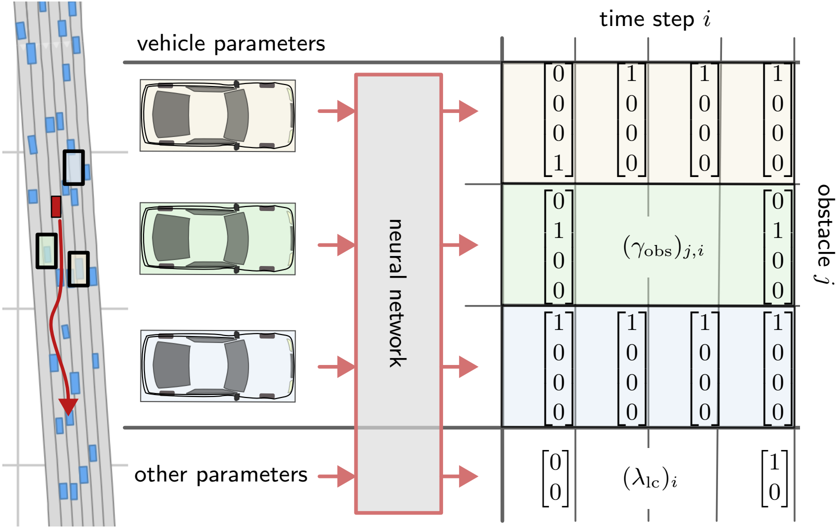
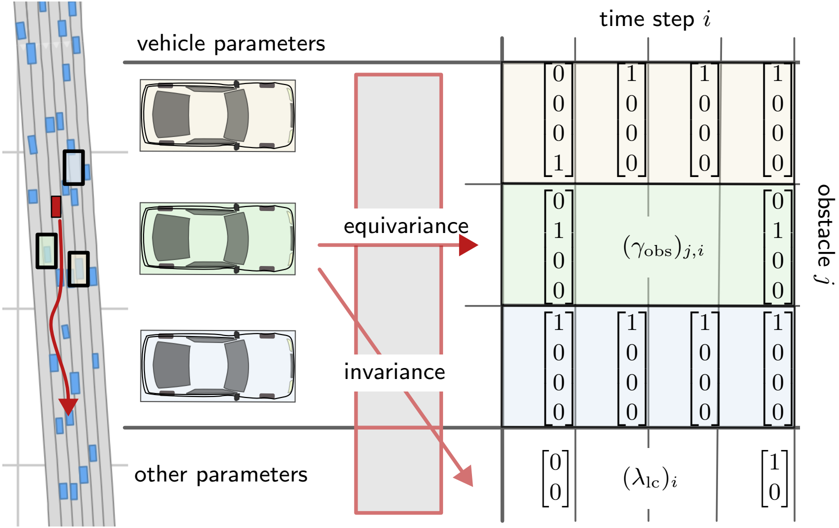
Why is invariance and equivariance interesting for our task?



time step i
vehicle parameters
neural network
obstacle j
$(\gamma_{\text{obs}})_{j,i}$
other parameters
$(\lambda_{\text{lc}})_i$



time step i
vehicle parameters
neural network
obstacle j
$(\gamma_{\mathrm{obs}})_{j,i}$
other parameters
$(\lambda_{\mathrm{lc}})_i$

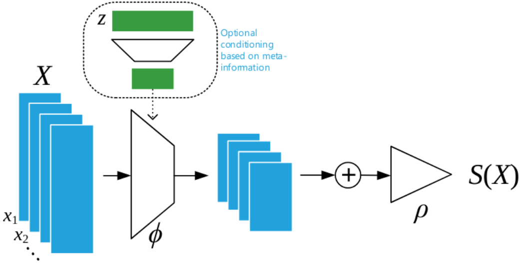


time step i
vehicle parameters
equivariance
invariance
$(\gamma_{\text{obs}})_{j,i}$
obstacle j
other parameters
$(\lambda_{\text{lc}})_i$

Number N_{p} of permutations of n elements is $N_{\mathrm{p}} = n!$
For 10 obstacles this would be $N_{\mathrm{p}} = 3628800$ different scenarios, while they all correspond to only one scenario

Let $x \in \mathbb{R}^D$ be features of a set element, $P \in \mathbb{R}^{M \times M}$ a permutation matrix and the matrix $X = (x_1, \dots, x_m)^\top \in \mathbb{R}^{M \times D}$ stacks the features as rows. A function $f(\cdot)$ is permutation invariant, iff $f(X) = f(PX)$. One permutation invariant function is

$$f(X) = \rho(\sum_{m=1}^{M} \phi(x_m))$$

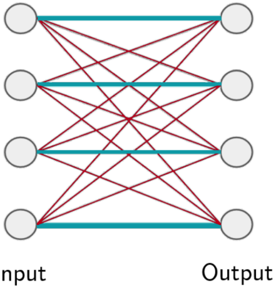


[3]Manzil Zaheer et al. "Deep Sets". In: *Advances in Neural Information Processing Systems*. Vol. 30. Curran Associates, Inc., 2017.

Permutation Equivariant Layers[4]

The function $f_\Theta(X) = \sigma(\Theta X)$, with $D = 1$, $\Theta \in \mathbb{R}^{M \times M}$, $X \in \mathbb{R}^M$ and $f_\Theta : \mathbb{R}^M \to \mathbb{R}^M$ is permutation equivariant iff all the off-diagonal elements of Θ are tied together and all the diagonal elements are equal as well. That is,

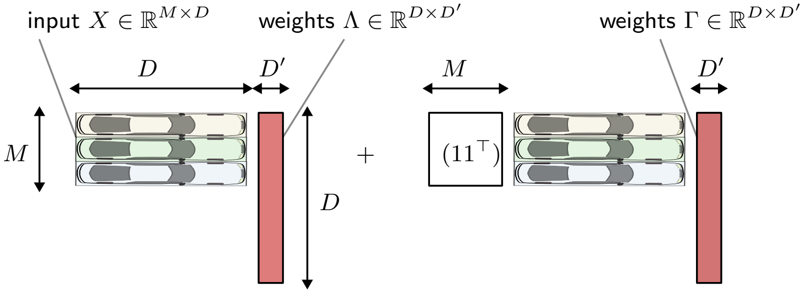
$$\Theta = \lambda I + \gamma(11^\top), \quad \lambda, \gamma \in \mathbb{R}, \quad 1^\top = [1, \dots, 1]^\top \in \mathbb{R}^M, \quad I \in \mathbb{R}^{M \times M} \text{ is identity}$$



[4] Zaheer et al., "Deep Sets".

This result can be easily extended to higher dimensions, i.e., D input and D' output channels. Then, $X \in \mathbb{R}^{M\times D}$, $y \in \mathbb{R}^{M\times D'}$, λ, γ become matrices $\Lambda, \Gamma \in \mathbb{R}^{D\times D'}$.

Layer function: $f(X) = \sigma(X\Lambda - (11^\top)X\Gamma)$



[5] Zaheer et al., "Deep Sets".

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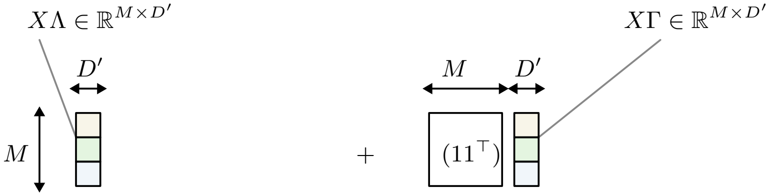

[6] Zaheer et al., "Deep Sets".

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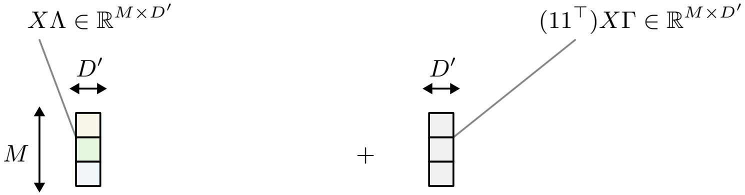
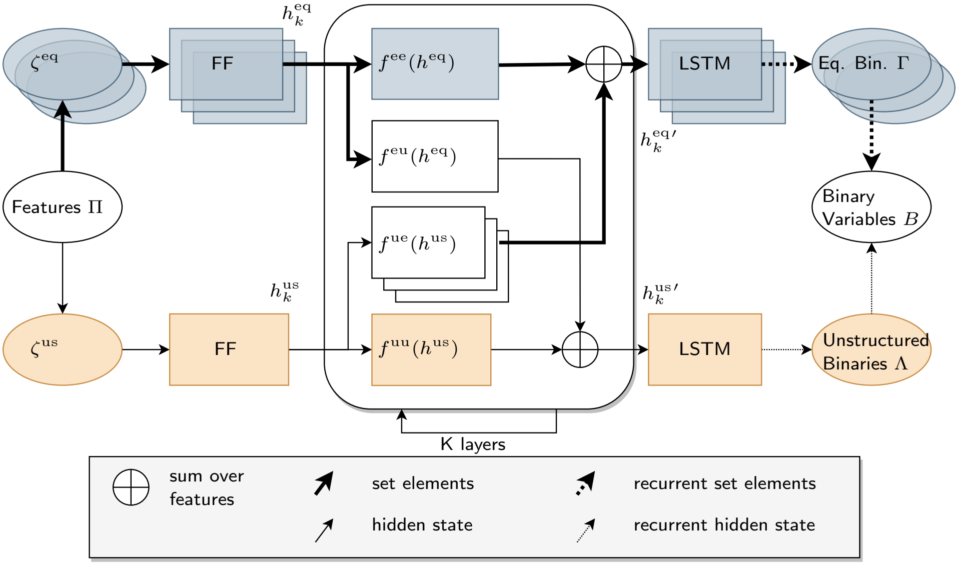


[7] Zaheer et al., "Deep Sets".

- The prediction is a time series of binary assignments $\rightarrow$ using a recurrent *decoder* to generate a time series[8]
- Allows for variable length predictions in addition to the variable number of obstacles

[8] Abhishek Cauligi et al. "PRISM: Recurrent Neural Networks and Presolve Methods for Fast Mixed-integer Optimal Control". In: *Proceedings of The 4th Annual Learning for Dynamics and Control Conference*. Vol. 168. Proceedings of Machine Learning Research. PMLR, 2022, pp. 34–46.

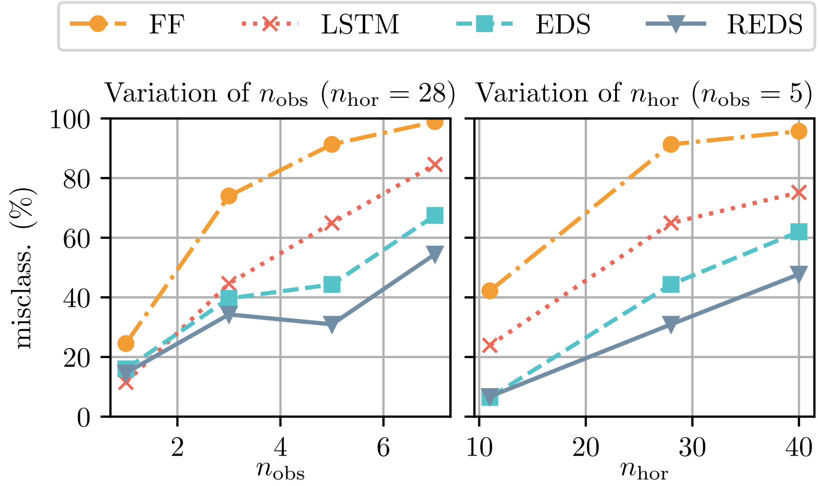
Final Recurrent Equivariant Deep Set Architecture



4. Additional Concepts

- Slacked QP: After predicting the binary variables, the remaining QP is solved with slacks on the fixed binary variables
- NN Ensemble: Several differently trained neural networks and slacked QPs are solved in parallel $\rightarrow$ lowest-cost solution is chosen
- Feasibility Projection: To enhance safety, an additional NLP is solved with nonlinear obstacle constraints to project possibly unsafe trajectories
- Lowest-level MPC: Lowest-level MPC tracks the planned trajectory

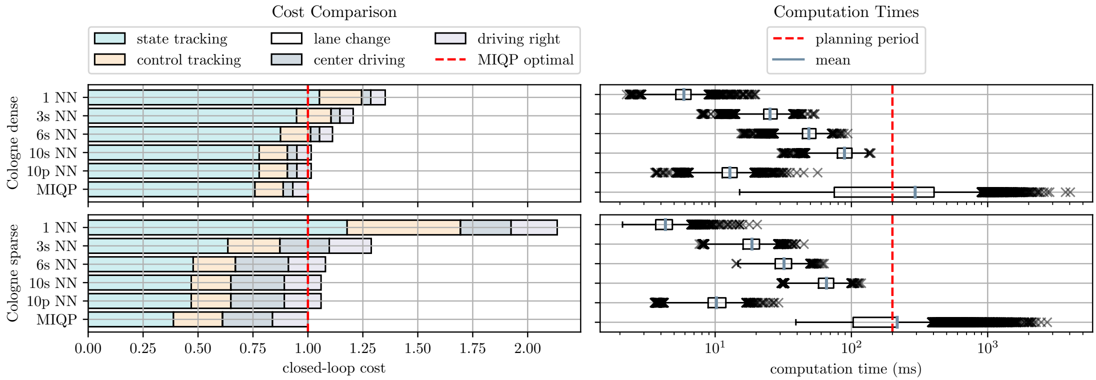
- Comparing the share of wrong predictions (misclassification) of all binary variables on test data set
- Architectures
  - Feed Forward (FF)
  - Long Short Term Memory (LSTM)
  - Equivariant and Invariant Deep Sets (EDS)
  - Equivariant, Invariant Layers and LSTM decoder (REDS)


FF
LSTM
EDS
REDS
Variation of n_{obs} ($n_{\mathrm{hor}} = 28$)
Variation of n_{hor} ($n_{\mathrm{obs}} = 5$)
misclass. (%)
100
80
60
40
20
0
2
4
6
n_{obs}
10
20
30
40
n_{hor}

- Comparing expert MIQP with proposed stack:
  - ensemble of $(1$ to $10)$ REDS networks for predictions of binaries
  - slacked QP
  - feasibility projector
- Both variants followed by a lowest-level NMPC tracking controller
- On randomized CommonRoad Cologne highway scenarios with SUMO backend

5. Results

Comparison in closed-loop simulations



Outlook

Interesting further work

- Diving deeper into geometric deep learning
  - Using geometric deep learning for other control systems tasks (e.g., exploiting invariances to other groups, such as Euclidean group)
  - Finding more generic layers for any MIQP (graph neural networks, transformers)
- More applications
  - Applying structure to large SUMO simulations for coordinating traffic
  - Multi-agent coordination of e.g., drones
- Improving the algorithm
  - Conditioned predictions along time axis to generate multiple prediction candidates
  - "Sandwiching" equivariant and recurrent layers

Thanks to the Coauthors!

Thank you for your attention!